# On Complex-Valued Equivariant Neural Networks for Radio Frequency Fingerprinting

Carter Brown, Enrico Mattei

Camila A. Ramirez

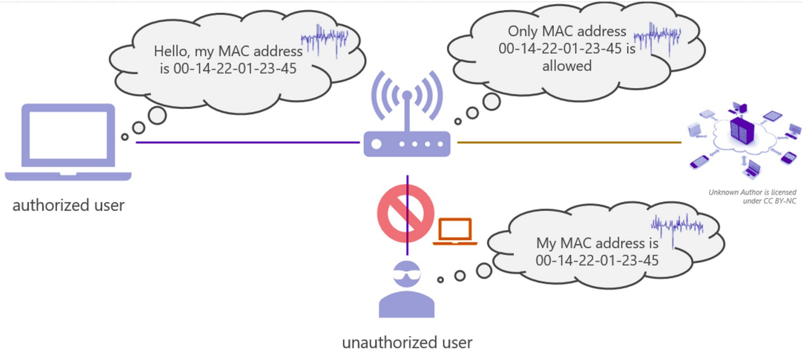
INTERNATIONAL COMPUTER SCIENCE INSTITUTE

EXPEDITION TECHNOLOGY

January 15, 2020

This research was developed with funding from the Defense Advanced Research Projects Agency (DARPA). The views, opinions and/or findings expressed are those of the author and should not be interpreted as representing the official views or policies of the Department of Defense or the U.S. Government.

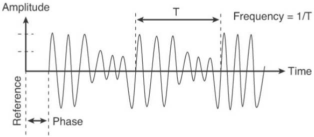
# Why finger print?



# Overview

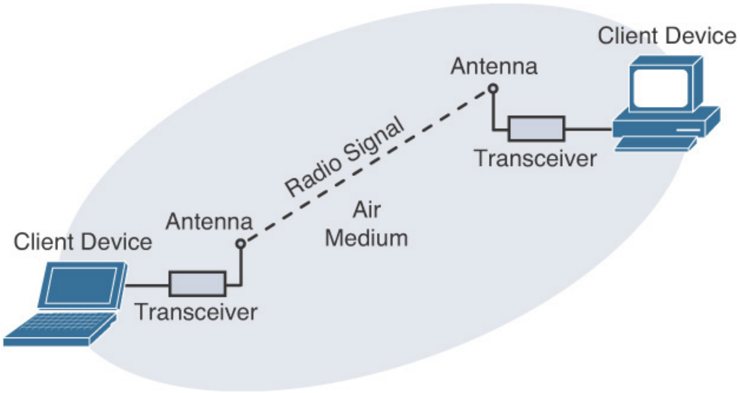
# Radio Waves

- Radio frequency (RF) signals carry information through the air
- Oscillate at a very high frequency
- Have amplitude, frequency, and phase elements (varied in time to represent information)



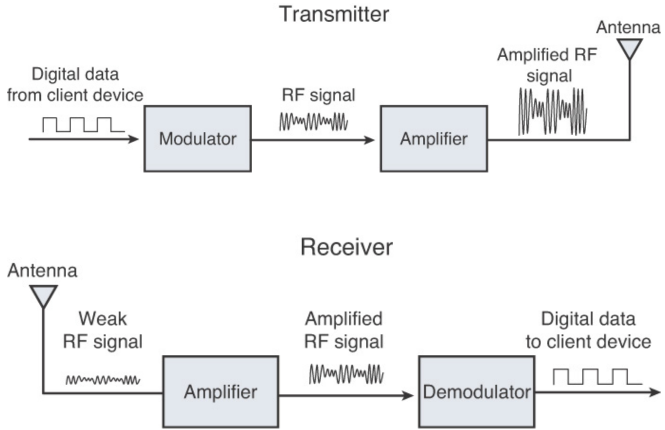

[1] J. Geier, (2015) *Designing and Deploying 802.11 Wireless Networks: A Practical Guide to Implementing 802.11n and 802.11ac Wireless Networks*, Cisco Press.

# Basic RF System




[1] J. Geier, (2015) *Designing and Deploying 802.11 Wireless Networks: A Practical Guide to Implementing 802.11n and 802.11ac Wireless Networks*, Cisco Press.

# RF Transceiver




[1] J. Geier, (2015) *Designing and Deploying 802.11 Wireless Networks: A Practical Guide to Implementing 802.11n and 802.11ac Wireless Networks*, Cisco Press.

# Why equivariance?

**Test data often contains perturbations not present in training data.**

Complex scalar multiplication:

- Attenuation (magnitude)
- Phase rotation

Complex vector multiplication:

- Channel and receiver effects

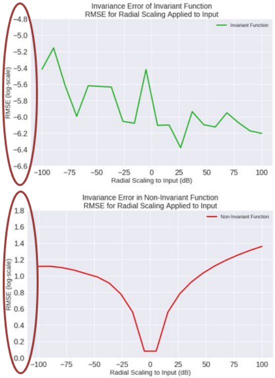
$\Rightarrow$ Degraded performance of learning algorithms

Can we learn representations of RF signals that are invariant with respect to perturbations of the input?

[2] R. Chakraborty, Y. Xing, S. Yu, SurReal: complex-valued deep learning as principled transformations on a rotational Lie group, arXiv preprint arXiv:1910.11334

# Approach

- Use polar coordinates $(r, \theta)$ to represent raw RF signals complex data
  $z \in \mathbb{C} \setminus 0 + 0i$
- Create pseudo-metric that is invariant to perturbations.
- Create convolutional layers that are equivariant perturbations.

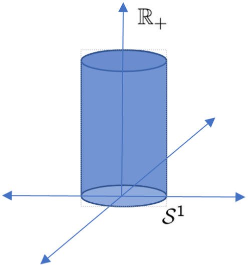



[3] S. Mallat, "Understanding deep convolutional networks, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences. Vol. 374, Issue 2065, 2016.

# Signals

Complex signals can be represented as $\mathcal{M} = \mathbb{R}_+ \times \mathcal{S}^1$

- $x \in \mathbb{R}_+$
- $\mathcal{S}^1$ is the unit circle
- $p \in \mathcal{S}^1$ is identified with $u(\theta) = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$



# Degradation

Attenuation and phase rotation can be defined as $\mathcal{G} = \mathbb{R}_+ \times \mathcal{SO}(2)$ with operation

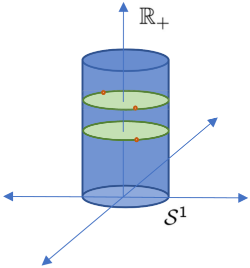
- $: g_1 = (r_1, R_1), g_2 = (r_2, R_2) \mapsto g_3 = (r_1 r_2, R_1 R_2)$

and $R_i = \begin{bmatrix} \cos(\phi_i) & -\sin(\phi_i) \\ \sin(\phi_i) & \cos(\phi_i) \end{bmatrix}$

# Group Action

Define an action $* : \mathcal{G} \times \mathcal{M} \rightarrow \mathcal{M}$ $(r, R) * (x, u(\theta)) \mapsto (rx, Ru(\theta))$

- Attenuation: scalar multiplication in the real component $x$
- Phase rotation: matrix multiplication $R$ on the unitary vector $u(\theta)$



# Pseudo-Metric

Define a function $d : \mathcal{M} \times \mathcal{M} \to \mathbb{R}$ as:

$$d^2(m_1, m_2) = \log^2\left(\frac{x_2}{x_1}\right) + \arccos^2\left(u(\theta_2)' u(\theta_1)\right)$$

We show $d(\cdot,\cdot)$ is invariant to the action $(\mathcal{G}, *)$ and use it to define a convolutional operation.

# Invariance

Consider a group element $g = (r, R) \in \mathcal{G}$ acting on $m_1, m_2 \in \mathcal{M}$:

$$d^2(g * m_1, g * m_2) = \log^2\left(\frac{rx_2}{rx_1}\right) + \arccos^2\left((Ru(\theta_2))' Ru(\theta_1)\right)$$

**$d^2(g * m_1, g * m_2) = d^2(m_1, m_2)$**

$$\begin{aligned}
&d^2(g * m_1, g * m_2) \\
&= \log^2\left(\frac{rx_2}{rx_1}\right) + \arccos^2\left((Ru(\theta_2))' Ru(\theta_1)\right) \\
&= \log^2\left(\frac{rx_2}{rx_1}\right) + \arccos^2\left(u(\theta_2)' R' Ru(\theta_1)\right) \\
&= \log^2\left(\frac{rx_2}{rx_1}\right) + \arccos^2\left(u(\theta_2)' u(\theta_1)\right) \\
&= d^2(m_1, m_2)
\end{aligned}$$

# Convolution

Define a convolutional map

$$\mathcal{F}\{w_i, m_i\} := \underset{m \in \mathcal{M}}{\arg\min} \sum_{i=1}^{N} w_i d^2(m_i, m)$$

$$\underset{m \in \mathcal{M}}{\arg\min} \sum_{i=1}^{N} w_i [\log^2 \left(\frac{x}{x_i}\right) + \arccos^2 \left(u(\theta)' u(\theta_i)\right)]$$

for $\{m_i\} \in \mathcal{M}$ some signal window and $\sum_{i=1}^{N} w_i = 1$.

# Convolution

$\mathcal{F}$ must be unique to show equivariance to the group action $(\mathcal{G}, *)$

$$g * \mathcal{F}(\{wi\}, \{m_i\}) = \mathcal{F}(\{wi\}, \{g * m_i\})$$

- Given some ordering, let $\{m_i\} \in \mathcal{M}$ be a pairwise ordered equidistant set. We call such sets *equidistant*.
- $\mathcal{F}$ is not properly defined for $\{m_i\}$ equidistant.
- Equidistant $\{m_i\}$ are closed under $(\mathcal{G}, *)$: rotating and scaling equidistant points remain equidistant.

We redefine $\mathcal{F}$ piecewise and thus satisfy uniqueness, allowing equivariance to hold.

# Piecewise Convolution

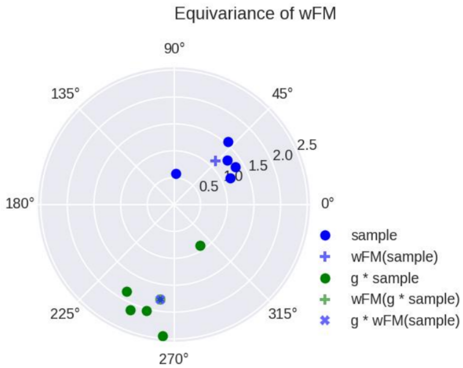
$$\mathcal{F}(\{w_i\},\{m_i\}) = \begin{cases} (\prod_{i=1}^{N} x_i^{w_i}, u(\min\{w_i\theta_i\}) & R = 0 \\ (\prod_{i=1}^{N} x_i^{w_i}, u(\hat{\theta})) & R > 0 \end{cases}$$

- $R = \sqrt{(\sum_{i=1}^{N} w_i \cos(\theta_i))^2 + (\sum_{i=1}^{N} w_i \sin(\theta_i))^2}$
- $\theta = \arctan \frac{\sum_{i=1}^{N} w_i \sin(\theta_i)}{\sum_{i=1}^{N} w_i \cos(\theta_i)}$
-

$$\hat{\theta} = \begin{cases} \theta & \sum_{i=1}^{N} w_i \cos(\theta_i) > 0 \text{ and } \sum_{i=1}^{N} w_i \sin(\theta_i) > 0 \\ \pi + \theta & \sum_{i=1}^{N} w_i \cos(\theta_i) < 0 \\ 2\pi + \theta & \sum_{i=1}^{N} w_i \cos(\theta_i) > 0 \text{ and } \sum_{i=1}^{N} w_i \sin(\theta_i) < 0 \end{cases}$$

# Equivariance of Convolution

$$g * \mathcal{F}(\{wi\}, \{m_i\}) = \mathcal{F}(\{wi\}, \{g * m_i\})$$



# Equivariance to Group Action

$$g * \mathcal{F}(\{w_i\}, \{m_i\}) = \mathcal{F}(\{w_i\}, \{g * m_i\})$$

**Setup:**

Let $m^* = \mathcal{F}(\{w_i\}, \{m_i\}) = \underset{m \in \mathcal{M}}{\arg\min} \sum w_i d^2(m_i, m)$

Let $\tilde{m} = \mathcal{F}(\{w_i\}, g * \{m_i\}) = \underset{m \in \mathcal{M}}{\arg\min} \sum w_i d^2(g * m_i, m)$

# Part 1

$\sum_i w_i d^2(g * m_i, \tilde{m}) = \sum_i w_i d^2(m_i, m^*)$

$\sum_i w_i d^2(g * m_i, \tilde{m})$
$= \min_{\tilde{\tilde{m}} \in g * \mathcal{M}} \sum_i w_i d^2(g * m_i, \tilde{\tilde{m}})$
$= \min_{g^{-1} * \tilde{\tilde{m}} \in g^{-1} g * \mathcal{M}} \sum_i w_i d^2(g^{-1} g * m_i, g^{-1} * \tilde{\tilde{m}})$
by invariance of distance metric
$= \min_{m^{**} \in \mathcal{M}} \sum_i w_i d^2(m_i, m^{**})$
$= \sum_i w_i d^2(m_i, m^*)$
by uniqueness of $\mathcal{F}$.

# Part 2

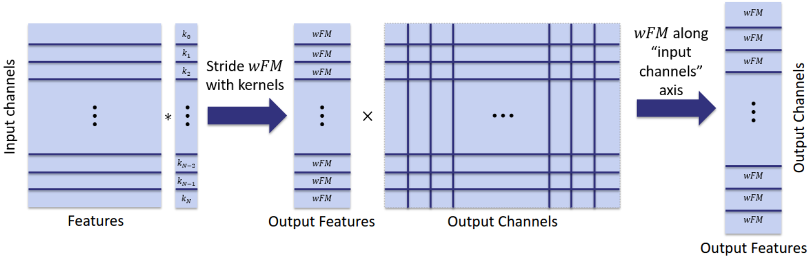
**$\tilde{m} = g * m^*$**

$\sum_i w_i d^2(g * m_i, \tilde{m})$
$= \sum_i w_i d^2(m_i, m^*)$
$= \sum_i w_i d^2(g * m_i, g * m^*)$
by invariance of distance metric.
$\Rightarrow \tilde{m} = g * m^*$, by uniqueness of $\mathcal{F}$.

$\therefore \mathcal{F}(\{w_i\}, \{g * m_i\}) = \tilde{m} = g * m^* = g * \mathcal{F}(\{w_i\}, \{m_i\}) \quad \square$

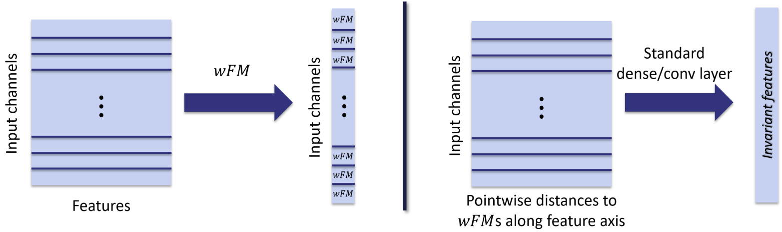
# PolarNet

Along each input channel axis, replace the dot product in a convolutional layer by $\mathcal{F}$.



# PolarNet

Invariance of $d^2(\cdot,\cdot)$ and equivariance of $\mathcal{F}(\cdot,\cdot)$ allows for the construction of an invariant layer as the distance from each input feature $m_i$ to $\mathcal{F}(\{w_i\},\{m_i\})$.

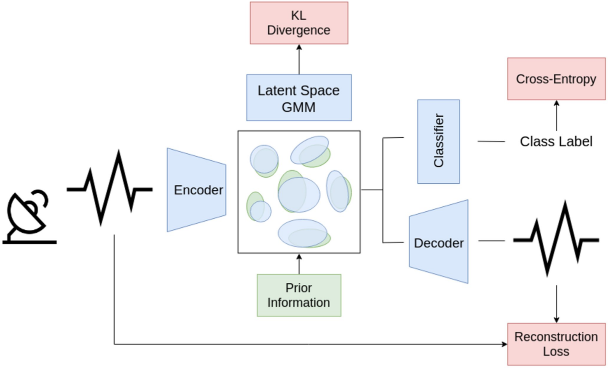


# Model Comparison

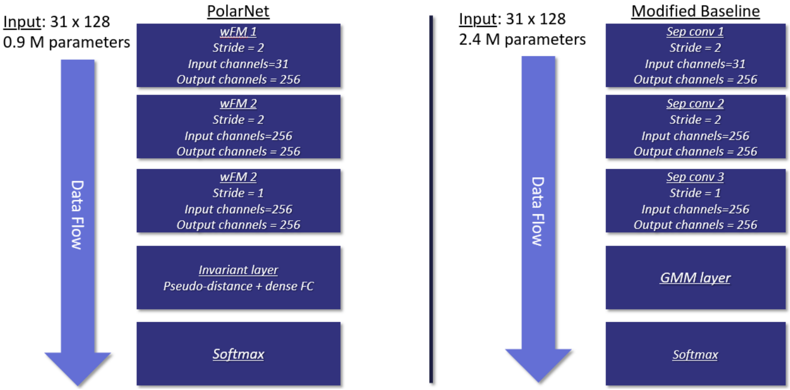
- PolarNet: polar complex representation $(r, \theta)$
- CartesianNet: Cartesian representation $(x, y)$
- Baseline model: our baseline model with default configuration
- Modified baseline model: baseline model with modifications to make the processing closer to PolarNet

# Model Comparison

## Baseline Model

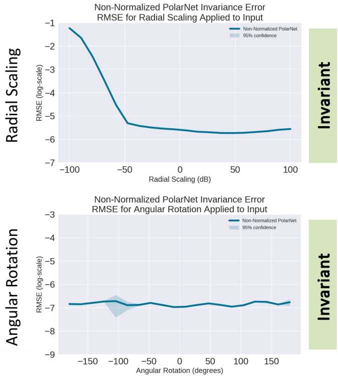


# Model Comparison

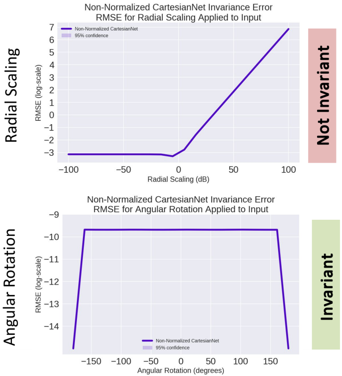


# Invariance Error: Non-Normalized

## PolarNet with Pseudo Metric



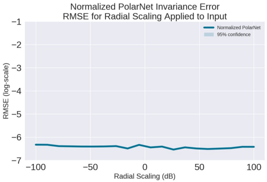

## CartesianNet with Euclidean Distance



# Invariance Error: Normalized

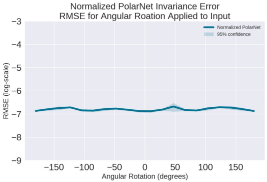
## PolarNet with Pseudo Metric

Radial Scaling




**Invariant**

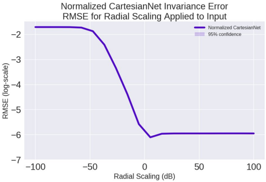
Angular Rotation




**Invariant**

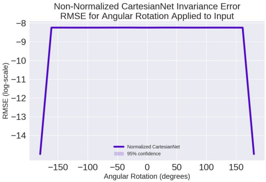
## CartesianNet with Euclidean Distance

Radial Scaling




**Invariant**

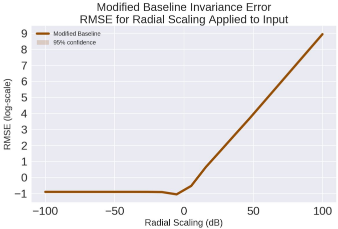
Angular Rotation




**Invariant**

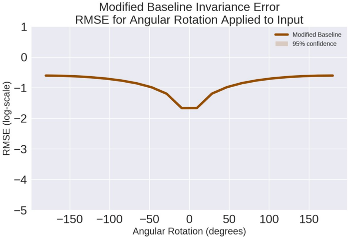
# Invariance Error: Non-Normalized

## Radial Scaling




**Not Invariant**

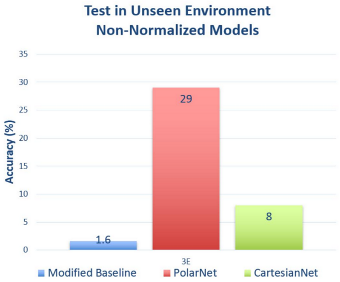
## Angular Rotation



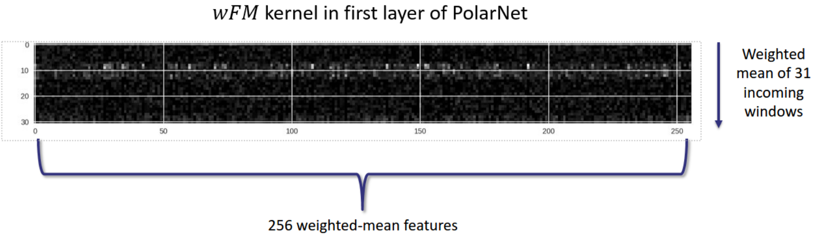

**Not Invariant**

# Performance: Non-normalized

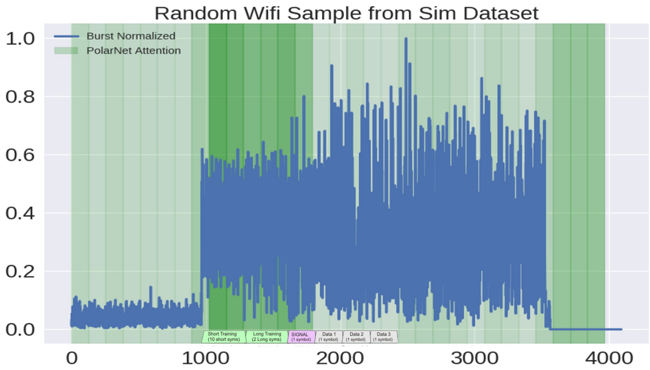
- No scale normalization applied in preprocessing, only mean centering
- PolarNet outperforms the other two models
- Comparing PolarNet to un-normalized networks shows the inherent invariance



# What is PolarNet learning?

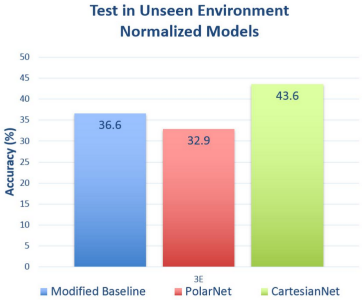


# What is PolarNet learning?



# Performance: Normalized

- Normalize bursts by max value in preprocessing
- Now all models invariant to radial scaling
- Modified Baseline model still not invariant to rotations
- CartesianNet has same invariances as PolarNet but is numerically more stable



# From Complex Scalar to Impulse Response

- The perturbation that we really want to be invariant to is convolution with an FIR filter.
- In frequency domain, convolution becomes element-wise multiplication.
- We can extend previous equations to filter perturbations.
- Thus construct a network invariant to filters (channels) in the frequency-domain.

# FIR Filter

$\mathcal{G} = \mathbb{R}_+^n \times \mathcal{T}^n$ with $\mathcal{T}^n \leq \mathcal{SO}(2n)$ the maximal torus subgroup, and operation $\bullet : g_1 = (\vec{r}_1, T_1), g_2 = (\vec{r}_2, T_2) \mapsto g_3 = (\vec{r}_1\vec{r}_2, T_1 T_2)$ where $\vec{r}_1\vec{r}$ is element-wise multiplication and $T \in \mathcal{T}^n$ is of the form:

$$T = \begin{bmatrix} R_1 & & & \\ & R_2 & & \\ & & \ddots & \\ & & & R_n \end{bmatrix}$$

with $R_i \in \mathcal{SO}(2)$.

# Signal

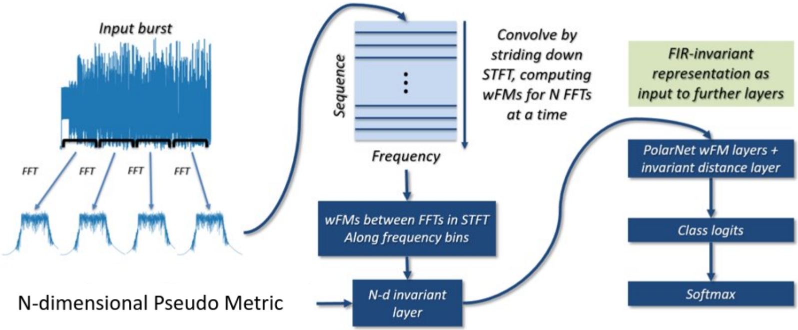
$\mathcal{M} = \mathbb{R}_+^n \times \mathcal{S}_1^1 \times \cdots \times \mathcal{S}_n^1$ with
$S = [\vec{u}_1(\theta_1) \cdots \vec{u}_n(\theta_n)] \in \mathcal{S}_1^1 \times \cdots \times \mathcal{S}_n^1$ a block diagonal matrix:

$$S = \begin{bmatrix} \begin{bmatrix}\cos(\theta_1)\\ \sin(\theta_1)\end{bmatrix} & & & \\ & \begin{bmatrix}\cos(\theta_2)\\ \sin(\theta_2)\end{bmatrix} & & \\ & & \ddots & \\ & & & \begin{bmatrix}\cos(\theta_n)\\ \sin(\theta_n)\end{bmatrix} \end{bmatrix}$$

# Degradation

Group action $* : \mathcal{G} \times \mathcal{M} \to \mathcal{M}$ as $(\vec{r}, T) * (\vec{x}, S) \mapsto (\vec{r}\vec{x}, TS)$.
Element-wise multiplication in the real component and block rotation $R_i|_T$ at each unitary vector $\vec{u}_i(\theta_i)|_S$.

# Channel-Invariant Network Architecture




N-dimensional Pseudo Metric

Thank you!